GeoVect S4   EXERCICE 1   1MR

```
plan := [[0, 5],[3, 8], [1/2, 5/2], [6/10, -4/5]]
```

$$\left[[0,5],[3,8],\left[\frac{1}{2},\frac{5}{2}\right],\left[\frac{3}{5},-\frac{4}{5}\right]\right]$$

```
espace := [[1, 2, -2], [1, 1, 1], [0, 3, -4]]
```

$$[[1,2,-2],[1,1,1],[0,3,-4]]$$

```
[matrix(plan[i]) $ i = 1..4];
[norm(matrix(plan[i]),2) $ i = 1..4]
```

$$\left[\begin{pmatrix}0\\5\end{pmatrix},\begin{pmatrix}3\\8\end{pmatrix},\begin{pmatrix}\frac{1}{2}\\\frac{5}{2}\end{pmatrix},\begin{pmatrix}\frac{3}{5}\\-\frac{4}{5}\end{pmatrix}\right]$$

a) $\left[5,\sqrt{73},\frac{\sqrt{2}\cdot\sqrt{13}}{2},1\right]$ ← vecteurs à deux composantes...

```
[matrix(espace[i]) $ i = 1..3];
[norm(matrix(espace[i]),2) $ i = 1..3]
```

$$\left[\begin{pmatrix}1\\2\\-2\end{pmatrix},\begin{pmatrix}1\\1\\1\end{pmatrix},\begin{pmatrix}0\\3\\-4\end{pmatrix}\right]$$

a) $[3,\sqrt{3},5]$ ← vecteurs à trois composantes...

```
vecS := matrix([-1/sqrt(5), 6/sqrt(45)]);
vecT := matrix([2/3, -1/3, 2/-3]);
norm(vecS, 2);
norm(vecS, 2);
```

$$\begin{pmatrix}-\frac{\sqrt{5}}{5}\\\frac{2\cdot\sqrt{5}}{5}\end{pmatrix}$$

$$\begin{pmatrix}\frac{2}{3}\\-\frac{1}{3}\\-\frac{2}{3}\end{pmatrix}$$

b) 1   Les vecteurs sont unitaires, car leur

1

norme vaut 1.

c)
```
vecA := matrix([3,4]);
vecB := matrix([12,-5]);
vecC := matrix([-6,0])
```

$$\begin{pmatrix}3\\4\end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 12 \\ -5 \end{pmatrix}$$

$$\begin{pmatrix} -6 \\ 0 \end{pmatrix}$$

```
norm(vecA,2)+norm(vecB,2)+norm(vecC,2)
```

24 $\quad \|\vec{a}\| + \|\vec{b}\| + \|\vec{c}\|$

```
norm(vecA+vecB+vecC,2)
```

$\sqrt{82}$ $\quad \|\vec{a} + \vec{b} + \vec{c}\|$

```
norm(-2*vecA,2)+2*norm(vecA,2)
```

20 $\quad \|-2\vec{a}\| + 2\|\vec{a}\|$

```
norm(vecA,2)*vecC
```

$\begin{pmatrix} -30 \\ 0 \end{pmatrix}$ $\quad \|\vec{a}\| \cdot \vec{c}$

```
1/norm(vecA,2)*vecA
```

$\begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix}$ $\quad \frac{1}{\|\vec{a}\|} \cdot \vec{a}$

$$\left\| \frac{1}{\|\vec{a}\|} \cdot \vec{a} \right\| = \frac{\|\vec{a}\|}{\|\vec{a}\|} = 1 \; !$$

d)

```
vecD := matrix([8, k-1]);
expand(norm(vecD, 2)^2)
```

$$\begin{pmatrix} 8 \\ k-1 \end{pmatrix}$$

$$\|\vec{d}\| = \sqrt{8^2 + (k-1)^2} = 10$$

$k \cdot \overline{k} - \overline{k} - k + 65$

```
solve(k^2-2*k+65=100,k)
```

$\Rightarrow 100 = 8^2 + (k-1)^2$

à résoudre

$\{-5, 7\}$

e)

```
vecU := matrix([2, 3]);
vecV := matrix([-2, 4])
```


$$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

```
expand(norm(vecU + m * vecV, 2)^2)
```

$$8 \cdot m + 8 \cdot \overline{m} + 20 \cdot m \cdot \overline{m} + 13$$

$$\| \vec{u} + m \cdot \vec{v} \| = \sqrt{20m^2 + 16m + 13}$$

```
solve(16*m + 20*m^2+13 = 82, m)
```

$$\left\{ -\frac{23}{10}, \frac{3}{2} \right\}$$

On doit résoudre : $20m^2 + 16m + 13 = 82$

EXERCICE 2

```
ptA := matrix([2,1,3]);
ptB := matrix([4,3,4]);
ptC := matrix([2,6,-9])
```

$$\vec{OA} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

$$\vec{OB} = \begin{pmatrix} 4 \\ 3 \\ 4 \end{pmatrix} \Rightarrow \vec{AB} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \quad \vec{AC} = \begin{pmatrix} 0 \\ 5 \\ -12 \end{pmatrix}$$

$$\vec{OC} = \begin{pmatrix} 2 \\ 6 \\ -9 \end{pmatrix} \quad \vec{BC} = \begin{pmatrix} -2 \\ 3 \\ -13 \end{pmatrix}$$

```
norm(ptB-ptA, 2)+norm(ptC-ptB, 2)+norm(ptA-ptC, 2)
```

$$\sqrt{182} + 16$$

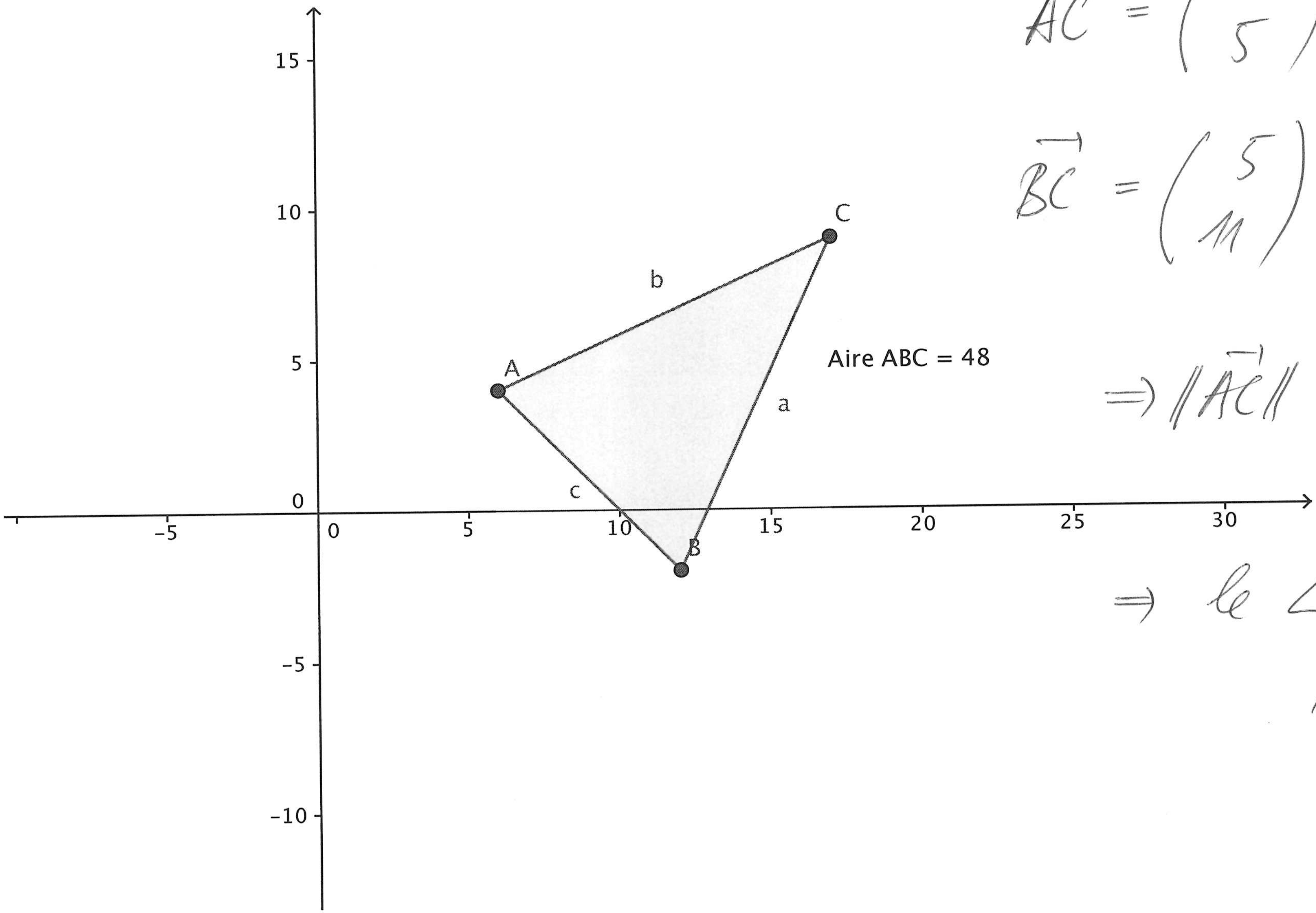
```
float(%)
```

29.49073756

$$\|\vec{AB}\| + \|\vec{BC}\| + \|\vec{AC}\| =$$

$$\sqrt{4+4+1} + \sqrt{25+144} + \sqrt{4+9+169}$$

Exercice 3
Série 4



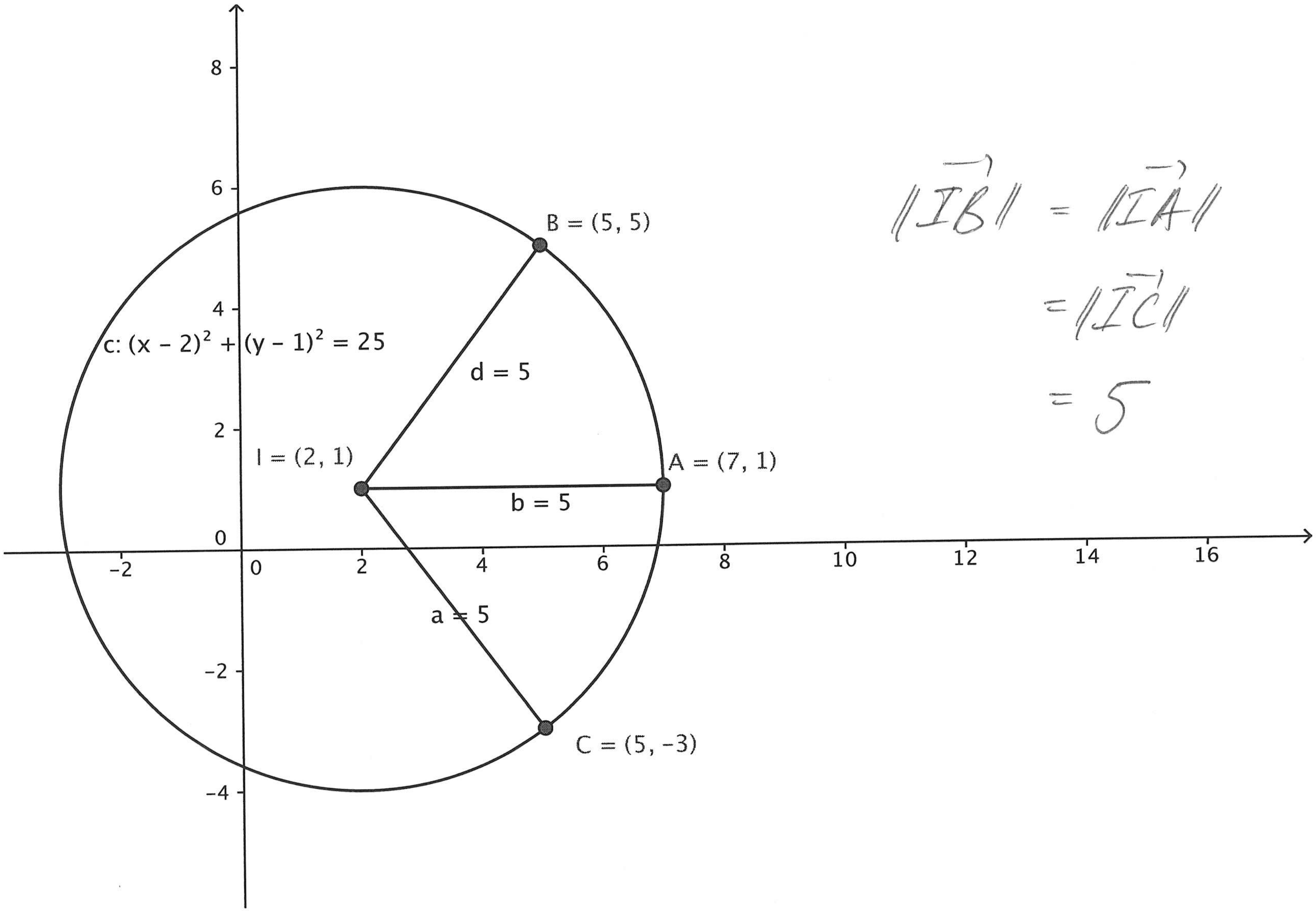
$$\vec{AC} = \begin{pmatrix} 11 \\ 5 \end{pmatrix}$$

$$\vec{BC} = \begin{pmatrix} 5 \\ 11 \end{pmatrix}$$

$$\Rightarrow \|\vec{AC}\| = \|\vec{BC}\|$$

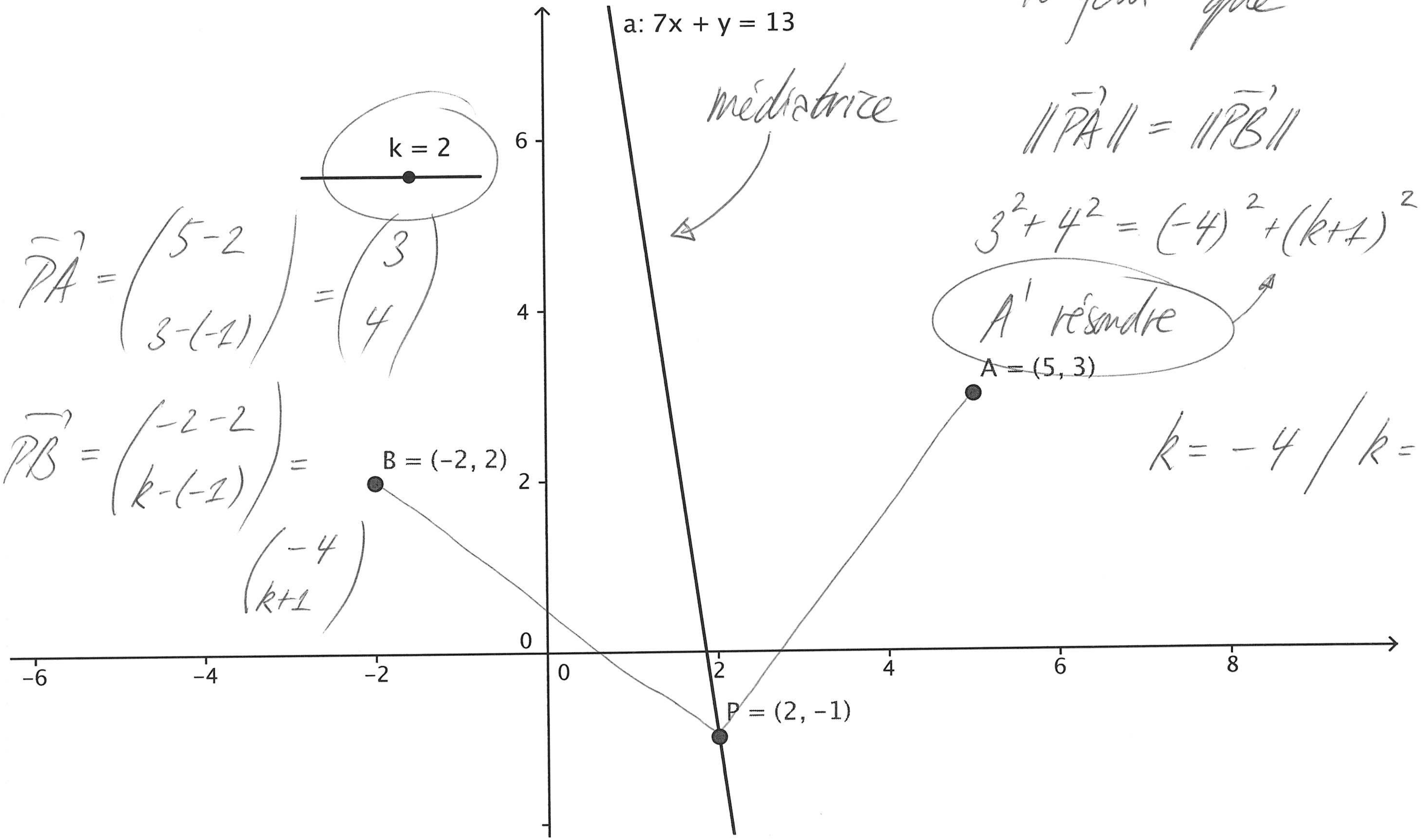
$\Rightarrow$ le $\triangle$ est isocèle

Exercice 4
Série 4



$$\|\vec{IB}\| = \|\vec{IA}\| = \|\vec{IC}\| = 5$$

Exercice 5
Série 4



médiatrice

$$\vec{PA} = \begin{pmatrix} 5-2 \\ 3-(-1) \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$\vec{PB} = \begin{pmatrix} -2-2 \\ k-(-1) \end{pmatrix} = \begin{pmatrix} -4 \\ k+1 \end{pmatrix}$$

Il faut que

$$\|\vec{PA}\| = \|\vec{PB}\|$$

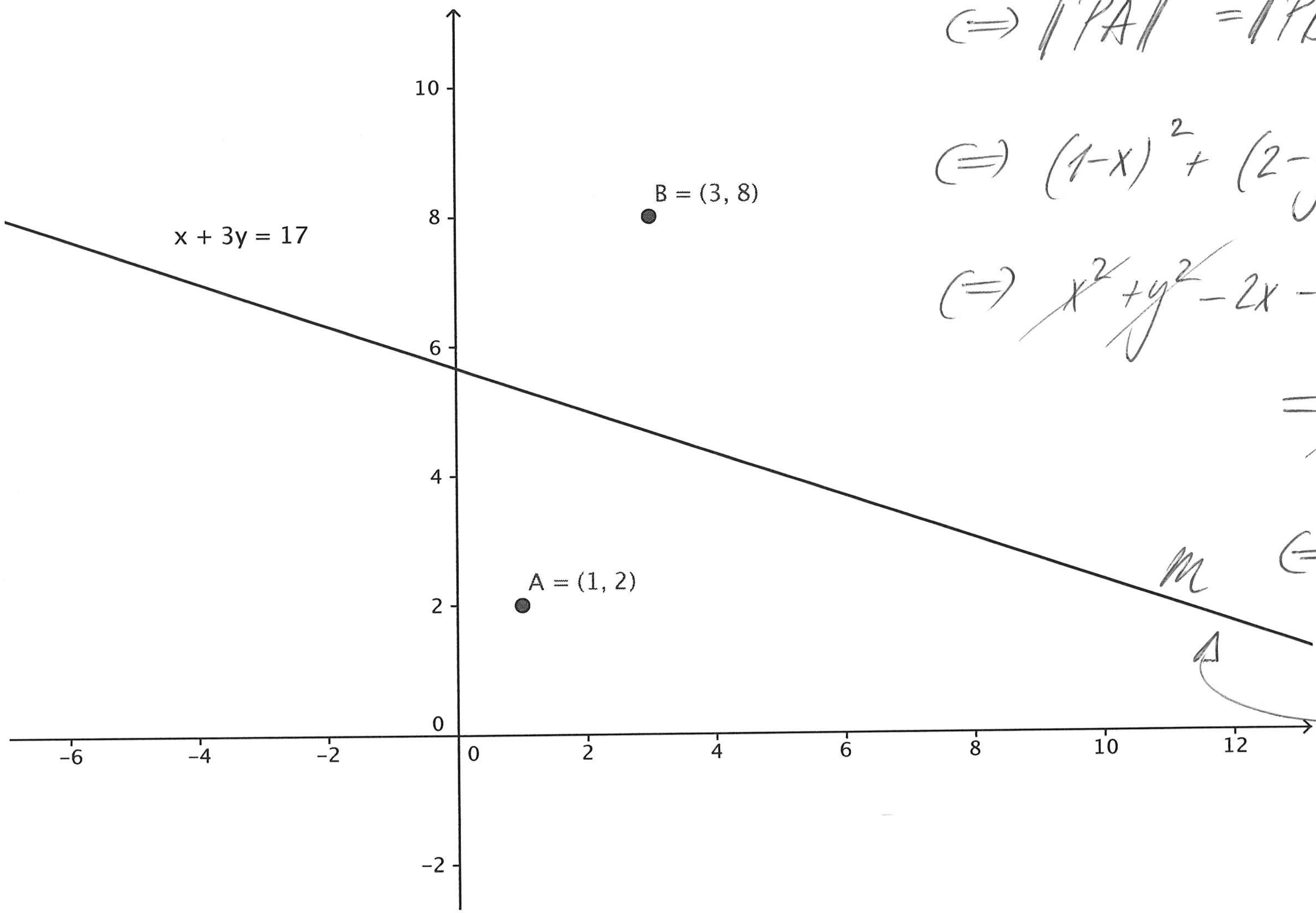
$$3^2 + 4^2 = (-4)^2 + (k+1)^2$$

A résoudre

$$k = -4 \ / \ k = 2$$

Exercice 6
Série 4

$P(x;y)$



P est sur m

$\Leftrightarrow \|\vec{PA}\|^2 = \|\vec{PB}\|^2$

$\Leftrightarrow (1-x)^2 + (2-y)^2 = (3-x)^2 + (8-y)^2$

$\Leftrightarrow \cancel{x^2} + \cancel{y^2} - 2x - 4y + 5$

$= \cancel{x^2} + \cancel{y^2} - 6x - 16y + 73$

$\Leftrightarrow x + 3y = 17$

Exercice 11

(a)

```
vecW :=  matrix([m, -2]);
vecZ :=  matrix([3, 5])
```

$$\begin{pmatrix} m \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

```
linalg::scalarProduct(vecW, vecZ)
```

$$3 \cdot m - 10$$

```
solve(3*m - 10 = 0, m)
```

$$\left\{\frac{10}{3}\right\}$$

(b)

```
vecA :=  matrix([1, 2, -3]);
vecB :=  matrix([2,1,4]);
vecC :=  matrix([6, -5, 0])
```

$$\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 6 \\ -5 \\ 0 \end{pmatrix}$$

```
solve(linalg::scalarProduct(vecA+k*vecB, vecC) = 0, k);
linalg::scalarProduct(vecA+k*vecB, vecC)
```

$$\left\{\frac{4}{7}\right\}$$

$$7 \cdot k - 4$$

(c)

```
vecU :=  matrix([1, 3]);
vecV :=  matrix([-3, 11]);
solve(linalg::scalarProduct(vecV-k*vecU, vecU) = 0, k);
linalg::scalarProduct(vecV-k*vecU, vecU);
vecV-3*vecU
```

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} -3 \\ 11 \end{pmatrix}$$

$\{3\}$

$30 - 10 \cdot k$

$$\begin{pmatrix} -6 \\ 2 \end{pmatrix}$$

(d)

```
vecE :=  matrix([7,a,b]);
vecF :=  matrix([4,3,8]);
vecG :=  matrix([-5,20,9]);
Eq_1 := linalg::scalarProduct(vecE, vecF);
Eq_2 := linalg::scalarProduct(vecE, vecG);
solve([Eq_1 =0, Eq_2 =0], [a,b]);
```

$$\begin{pmatrix} 7 \\ a \\ b \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$$

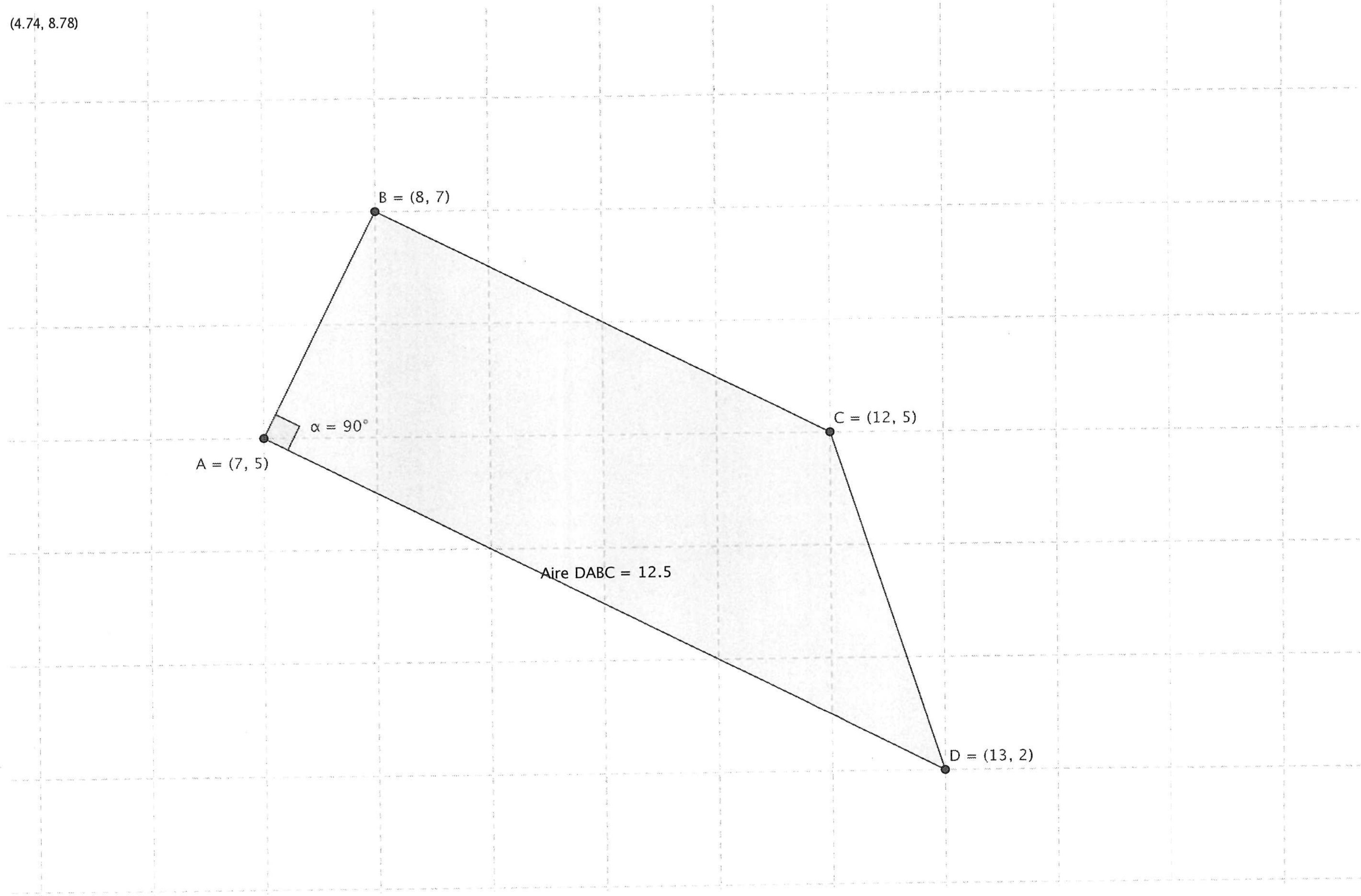
$$\begin{pmatrix} -5 \\ 20 \\ 9 \end{pmatrix}$$

$3 \cdot a + 8 \cdot b + 28$

$20 \cdot a + 9 \cdot b - 35$

$\{[a = 4, b = -5]\}$

Exercice 12
Série 4



Exercice 14

```
vecA :=  matrix([3,4]);
vecB :=  matrix([5,-1]);
vecC :=  matrix([7,1]);
vecD :=  matrix([0,3]);
```

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 5 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

(a)

```
linalg::scalarProduct(vecA, 7*vecB + vecC)
```

$$102$$

(b)

```
linalg::scalarProduct(vecA, vecB) * vecB
```

$$\begin{pmatrix} 55 \\ -11 \end{pmatrix}$$

(c)

```
linalg::scalarProduct(vecA, vecC) + linalg::scalarProduct(vecC, vecD)
```

$$28$$

(d)

```
linalg::scalarProduct(vecA+vecB, vecC-vecD)
```

$$50$$

(e)

```
norm(vecD, 2) * linalg::scalarProduct(vecA, vecD)
```

$$36$$

(f)

```
vecA + linalg::scalarProduct(vecB, vecC)
```

$$\begin{pmatrix} 37 \\ 4 \end{pmatrix}$$

Exercice 16

```
pointA := matrix([-2,4]);
pointB := matrix([1, -2]);
pointC := matrix([x,x]);
```

$$\begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ x \end{pmatrix}$$

```
vecAB := pointB - pointA;
vecBC := pointC - pointB;
vecCA := pointA -pointC;
```

$$\begin{pmatrix} 3 \\ -6 \end{pmatrix}$$

$$\begin{pmatrix} x-1 \\ x+2 \end{pmatrix}$$

$$\begin{pmatrix} -x-2 \\ 4-x \end{pmatrix}$$

(a)

```
solve(linalg::scalarProduct(vecAB, vecCA) = 0, x);
```

$$\{10\}$$

(b)

```
solve(linalg::scalarProduct(vecAB, vecBC) = 0, x);
```

$$\{-5\}$$

(c)

```
solve(linalg::scalarProduct(vecBC, vecCA) = 0, x);
```

$$\left\{-2, \frac{5}{2}\right\}$$